W.Q.8 Prove that (1)

B.Sc Math (H) part-1st
paper-1st, Group A
Summation of series
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$$\frac{\sin\theta - \sin 2\theta + \sin 3\theta - \dots \text{ to } n \text{ terms}}{\cos\theta - \cos 2\theta + \cos 3\theta - \dots \text{ to } n \text{ terms}} = \tan\left\{\frac{n+1}{2}(\pi+\theta)\right\}$$

$$\text{L.H.S} = \frac{\sin\theta + \sin(\pi+2\theta) + \sin(2\pi+3\theta) + \dots \text{ to } n \text{ terms}}{\cos\theta + \cos(\pi+2\theta) + \cos(2\pi+3\theta) + \dots \text{ to } n \text{ terms}}$$

$$= \frac{\dfrac{\sin n\frac{\pi+\theta}{2}}{\sin\frac{\pi+\theta}{2}} \cdot \sin\left\{\theta + (n-1)\frac{\pi+\theta}{2}\right\}}{\dfrac{\sin n\left(\frac{\pi+\theta}{2}\right)}{\sin\left(\frac{\pi+\theta}{2}\right)} \cos\left\{\theta + (n-1)\frac{\pi+\theta}{2}\right\}}$$

$$= \frac{\sin \frac{2\theta + (n-1)(\pi+\theta)}{2}}{\cos \frac{2\theta + (n-1)(\pi+\theta)}{2}}$$

$$= \tan\left\{\frac{2\theta + n(\pi+\theta) - \pi - \theta}{2}\right\}$$

$$= \tan\left\{\frac{2\theta + n\pi + n\theta - \pi - \theta}{2}\right\}$$

$$= \tan \frac{(\theta + n\theta) + n\pi - \pi}{2}$$

$$= \tan\left(\frac{\pi}{1} + \frac{\theta + n\theta + n\pi - \pi}{2}\right\} \quad \left[\because \tan(\pi + A) = \tan A \text{ Ho...} \right]$$

$$= \tan \theta + n\theta + n\pi + \pi$$

$$= \tan\left\{\frac{2\pi + \theta + n\theta + n\pi - \pi}{2}\right\}$$

$$= \tan \frac{\theta + n\theta + n\pi + \pi}{2}$$

②

$$= \tan \frac{\theta(n+1) + \pi(n+1)}{2}$$

$$= \tan \frac{(n+1)(\pi+\theta)}{2} \quad \text{Ans}$$

<u>W.Q.</u> If y be the exterior angle of a regular polygon of n sides show that

$\cos x + \cos(x+y) + \cos(x+2y) + \ldots$ to n terms $= 0$

[As y is the exterior angle of a regular polygon of n sides

सभी भुजाएँ बराबर हो
उसे हम polygon कहते हैं।
regular polygon
का बाहरी निकाय कोण
को कहते

$\therefore y = \frac{2\pi}{n}$ (R.) (Remember)

polygon है

$\therefore n = \frac{2\pi}{y}$

$$L.H.S = \frac{\sin \frac{ny}{2}}{\sin \frac{y}{2}} \cos\left\{x + (n-1)\frac{y}{2}\right\}$$

$$= \frac{\sin\left(\frac{2\pi}{y} \times \frac{y}{2}\right)}{\sin \frac{y}{2}} \cos\left\{x + (n-1)\frac{y}{2}\right\}$$

$$= \frac{\sin \pi}{\sin \frac{y}{2}} \cos\left\{x + (n-1)\frac{y}{2}\right\} \quad \because \sin\pi = 0$$

$$= \frac{0}{\sin \frac{y}{2}} \cos\left\{x + (n-1)\frac{y}{2}\right\} = 0 \quad \text{proved}$$